Spectral graph theory and its application in virus spread in a computer network

Abstract: The survey encompasses an explanation of particular topics from the concept of the graph spectra independently of the area of computer science in which they're used. Eigenvalues and eigenvectors of numerous graph matrices seem in numerous papers on various topics relevant to statistics and communication technologies. Particularly, we survey applications in modelling and searching internet, in computer vision, data mining, multiprocessor structures, statistical databases, and numerous different regions.

Introduction:

In this project we shall deliver an assessment of parts of theory of graph spectra that are useful in computer sciences. Here we will provide a survey of applications of the theory of graph spectra to computer science. Spectral graph theory represents these days a very useful mathematical tool in the information technology, enrolled from text search and retrieval, and for refining predictive-evaluation systems. Spectral graph principle is based on linear algebra, and its properly-evolved part, the matrix theory.Graph theory just facilitates in expertise structural relations in big data sets which includes even billion of elements (called vertices, or nodes), and relations among them represented by links (called edges in undirected case, or arcs in any other case).

In this assignment we are not giving a survey on applications of matrices in computer science, or on applications of graphs in computer science. We need to survey applications of the theory of graph spectra (or of spectral graph theory) in computer science.

Spectral graph theory is a mathematical principle in which linear algebra and graph theory meet. For any graph matrix M we can construct a spectral graph principle in which graphs are studied through eigenvalues of the matrix M. This theory is known as M-principle. A good way to keep away from confusion, to any notion in this theory a prefix M- could be added (e.g., M-eigenvalues). Frequently used graph matrices are the adjacency matrix A, the Laplacian $L = D - A$ and the sign less Laplacian $Q = D + A$, in which D is a diagonal matrix of vertex degrees. The spectral graph idea includes all these unique theories together with interaction tools. It became identified in approximately the last ten years that graph spectra have several critical applications in computer sciences. Graph spectra appear within the literature in internet technologies, computer vision, pattern recognition, data mining, multiprocessor systems, and statistical databases and in many other areas. There are heaps of such papers.

One should be stated that spectra of numerous graph matrices appear in applications. The adjacency matrix and Laplacian appear most often but also the sign less Laplacian in addition to normalized versions of those matrices. Incidence, distance and other matrices can be discovered as well. From time to time the considerations pass from graph matrices to popular ones; equivalently, weighted graphs appear rather than graphs. In some cases we stumble upon digraphs and hyper-graphs as properly.

It could be observed that during maximum cases not only the eigenvalues but also the eigenvectors of relevant graph matrices appear in applications.

Several researchers in computer science declare that spectral graph theory is (one of) their scientific field(s). Notice that mathematicians usually communicate of the theory of graph spectra or just graph spectra.

The books [1], [2] contain each a chapter on applications of graph eigenvalues. The book [3] additionally consists of a chapter on applications. There are sections on Physics, Chemistry, computer Sciences and mathematics itself.

In keeping with its Preface, the motive of the e-book [4] is to attract the eye of mathematical community to swiftly growing packages of the theory of graph spectra. Besides classical and well documented applications to Chemistry and Physics, we are witnesses of the appearance of graph eigenvalues in computer science in various investigations. There also are applications in several other fields like Biology, Geography, Economics and Social Sciences. The book [4] contains 5 chapters: an introductory chapter with a survey of applications via consultant examplesand 4 case studies (one in computer science and three in Chemistry). The book [5] is an extended and progressed edition of the preceding one.

The introductory text [6] offers an advent to the theory of graph spectra and a brief survey of applications of graph spectra together with a few decided on bibliographies on applications. we have stated applications to Chemistry, Physics, computer Sciences and mathematics itself.

Relating to the book [1] as “the current standard work on algebraic graph concept”, Van Mieghem gave in his book [7] a twenty page appendix on graph spectra, hence pointing out the importance of this difficulty for communications networks and structures. The paper [8] is a tutorial on the basic facts of the theory of graph spectra and its applications in computer science delivered at the forty eighth Annual IEEE Symposium on Foundations of computer science.

The two of us (D. C. and S. k. S.) have posted a survey [9] of the programs of graph spectra in computer science. we have diagnosed numerous applications inside the following branches of computer science:

1. Expanders and combinatorial optimization,

2. Complex networks and the Internet topology,

3. Data mining,

4. Computer vision and pattern recognition,

5. Internet search,

6. Load balancing and multiprocessor interconnection networks,

7. Anti-virus protection versus spread of knowledge,

8. Statistical databases and social networks,

9. Quantum computing.

Subsequently, we have also become aware of applications in

10. Bioinformatics,

11. Coding theory,

12. Control theory.

This class of numerous packages carries some overlapping in the classified material. for example, methods of data mining (mainly, spectral graph clustering) seem in computer vision, social networks and internet search while several problems of combinatorial optimization are relevant for data mining (e.g., in clustering). Since methods of computer science are found in all branches of science, applications of graph spectral techniques to computer science are transferred to nearly all branches of science (telecommunications, electrical engineering, biology, chemistry, geography, social sciences, and so forth.). Sometimes by using the usage of the adjective "computational" you may denote those components of particular sciences which overlap with computer science (e.g., computational biology, computational chemistry, and many others.). In this sense one can speak of computer sciences as we have put in the title of this paper.

Of course, graph spectra seem in computer science since graphs for themselves are relevant. The principle gain of using graph spectra comes from the fact that eigenvalues and eigenvectors of several graph matrices can be quickly computed (computational complexity is O (n^3) where n is the number of vertices). However, spectral graph parameters contain a variety of records on the worldwide and neighbourhood). This includes some information on graph parameters that, in general, are computed by means of exponential algorithms (e.g. chromatic number, the size of maximal clique, etc.). For example, computing the chromatic wide variety of a graph with a few thousands vertices is a difficult venture even as eigenvalues and eigenvectors can be computed in a few seconds.

Graphs which can be treated in computer sciences using graph spectra typically represent both some physical networks (computer network, net, biological network, etc.) or data structures (documents in a database, indexing structure, and so on.) within the first case the graphs usually have a great number of vertices (thousands or millions) and they're called complex networks while in the second case graphs are of small dimensions.

A. Expanders and combinatorialoptimization

One of the oldest applications (from 1970's) of graph eigenvalues in Computer Science is related to graphs called *expanders*. Informally, we shall say that a graph has good expanding propertie*s*if each subset of the vertex set of small cardinality has a set of neighbours of large cardinality. Ex- panders and some related graphs (called *enlargers*, *magnifiers*, *concentrators* and *super-concentrators*, just to mention some specific terms) appear in treatment of several problems in Computer Science (for example, communication networks, error-correcting codes, optimizing memory space, computing functions, sorting algorithms, etc.). Expanders can be constructed from graphs with a small second largest eigenvalue in modulus. Such class of graphs includes the so called *Ramanujan graphs*. Paper [10] is one of the most important papers concerning Ramanujangraphs.

Expanders are associated with some troubles of combinatorial optimization. More generally, several algorithms of combinatorial optimization are considered as part of computer science.

A sensor network consists of spatially distributed sensors (with limited capacities) and links connecting them. One of the basic problems with these networks is to design a topology (connection graph) thatmaximizestheratio $\nu_2 / \nu_n$where ν_n is the largest while ν_2 the second smallest eigenvalue of the graph Laplacian. This eigenvalue is called algebraic connectivity of the graph and was introduced by M. Fiedler [21].

The larger is this ratio, the faster is the convergence speed of the decision fusion algorithm, and as a consequence better the overall performance of the network. The same ratio is applicable for the process of synchronization in complicated networks [12]. In [25], it was pointed that (non-bipartite) Ramanujan graphs are good candidates for preferred topologies. There are many other troubles in sensor networks where the equipment from the combinatorial optimization and spectral graph concept can assist, say in solving partitioning, project, routing and scheduling issues.

B. Complex networks and the Internet

Complex networks is a not unusual name for various real networks which might be provided by means of graphs with an enormously great number of vertices. Right here belong internet graphs, phonographs, e-mail graphs, social networks and many other. Regardless of their variety such networks share a few not unusual properties. Several models of random graphs have been used to explain complicated networks inclusive of the classical Erd¨os-R´enyi model in which we've a consistent possibility for the existence of each edge.

There are models where given degree distribution is realized. Main characteristic of complex networks is the degree and eigenvalue distribution. Both distributions obey a power law form $x^{-\beta}$ for a positive constant βIn particular, if $n_k$denotes the number of vertices of degree, then asymptotically $n_k = ak^{-\beta}$ for some constant a.
It was conjectured in [13] that in networks with degree power law the largest eigenvalues of the adjacency matrix have also a power law distribution. That was proved under some conditions in [13].

The power law for eigenvalues can be formulated in the following way. Let $\lambda_1, \lambda_2 \ldots$.be non-increasing sequence of eigenvalues of the adjacency matrix, then asymptotically $\lambda_k = ak^{-\gamma}$ for some constant a and positive γ

The Internet is a collection of thousands of local networks (Autonomous Systems) of computers (hosts and routers). Autonomous Systems are linked by a common set of protocols which enable communication and allow the use of services located at any of the other Autonomous Systems.

The whole internet, or a part of it, may be represented by means of a graph in which the vertices correspond to hosts and routers while the edges correspond to physical connections between them. In any other representations the vertices correspond to self-reliant structures and the rims to the hyperlinks. Reading and modeling internet topology (i.e. the structure) is vital for protocol performance assessment and simulation of a ramification of community issues. The main theoretical fashions of the net use the concepts of complicated networks and, specifically, energy laws for stages and eigenvalues.

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as normalized versions of these matrices. Occurrence, distance and other matrices can be determined as well. Every now and then the issues flow from graph matrices to widespread ones; equivalently, weighted graphs seem as opposed to graphs. In a few instances we come across digraphs and hyper-graphs and corresponding matrices as well.

One can notice that not only the eigenvalues but also the eigenvectors of relevant graph matrices appear in applications in most cases. In many papers the normalized Laplacian matrix $\hat{L}= D^{\frac{-1}{2}} L D^{\frac{-1}{2}} = D^{\frac{-1}{2}} L D^{\frac{-1}{2}}$ appears. This matrix has 1's on the diagonal, and at an off-diagonal position (i, j) the entry is equal to 0 for non-adjacent and $-1/\sqrt{d_i d_j}$ for adjacent vertices i,j of degree d_i, d_j. The spectrum of $\hat{L}$ belongs to the interval [0, 2] independently of the number of vertices.

For non-trivial connected graphs the matrices $D^{-1}A$ and $(2D)^{-1}Q = (2D)^{-1}(D + A) = \frac{1}{2}(I + D^{-1}A)$ are transition matrices of Markovchains for random and lazy random walks.

Note that the normalized Laplacian matrix $\hat{L} = D^{\frac{-1}{2}} L D^{\frac{-1}{2}} = D^{\frac{-1}{2}}(D-A) D^{\frac{-1}{2}} = I - D^{\frac{-1}{2}} A D^{\frac{-1}{2}}$ *and the* normalized sign less Laplacian matrix $\hat{Q} = D^{\frac{-1}{2}} Q D^{\frac{-1}{2}} = D^{\frac{-1}{2}}(D + A) D^{\frac{-1}{2}} = I + D^{\frac{-1}{2}} A D^{\frac{-1}{2}}$ are connected by the relation $\hat{Q}= -\hat{L}+2I$ this means the $\hat{Q}$ theory is simply reduced to $\hat{L}$ theory. A similar statement hold for the matrix $D^{\frac{-1}{2}} A D^{\frac{-1}{2}}$

Empirical studies of the Internet topology have been conducted in many papers using the normalized Laplacian matrix $\hat{L} = D^{\frac{-1}{2}}(D-A) D^{\frac{-1}{2}}$. This matrix has 1's on the diagonal, and at an off-diagonal position (i,j) the entry is equal to 0 for non-adjacent and $-1/\sqrt{d_i d_j}$ for adjacent vertices belongs to the interval [0,2] independently of the number of vertices. The book [14] is devoted to the normalized Laplacian.

The eigenvalues γ_i= 1,2……nof $\hat{L}$ in non-decreasingorder can be represented by points $(\frac{i-1}{n-1}, \gamma_i)$ in the region[0,1] X[0,2] and can be approximated by a continuouscurve. It was noticed in [15] that this curve is practicallythe same during the time for several networks in spite of theincreasing number of vertices and edges of the correspondinggraph. Therefore the authors consider the spectrum of $\hat{L}$ as afingerprint of the corresponding network topology. Aspectrally based measure of similarity between networkshas been introduced in [23], and applied to Internet topologyanalysis.

C. Data mining

Data mining is defined as a technique used to extract usable data from a larger set of any raw data. It implies analysing statistics patterns in large batches of facts using one or more software. Data mining has applications in multiple fields, like science and research. As an application of data mining, businesses can learn more approximately their clients and expand more effective strategies related to various commercial enterprise functions and in turn leverage resources in a more optimal and insightful way. This allows businesses be closer to their goal and make higher selections. Data mining involves effective data collection and warehousing in addition to computer processing. For segmenting the data and evaluating the possibility of future events, data mining makes use of sophisticated mathematical algorithms. Data mining is also called knowledge Discovery in data (KDD).

A description of spectral clustering techniques is given in the tutorial [15]. Here we expect that the data are represented by using a graph. We shall present a set of rules for graph clustering which is primarily based on the Laplacian matrix of a graph.

Let G be a connected graph on n vertices. Eigenvalues in non-decreasing order and corresponding orthonormal eigenvectors of the Laplacian $\hat{L}$=D-Aof G are denoted by v_1= 0v_2……..$v_n$andu_1, u_2,…….u_n, respectively.

In order to construct k clusters in a graph we form an nXkmatrix u containing the vectors u1,u2…….uk as columns. In this way we have constructed a geometric representation G of G in the k-dimensional space Rk: we just take rows of u as point coordinates representing the vertices of G. Edges are straight line segments between the corresponding points. Now classical clustering methods (say k-means algorithm) should be applied to this new graph presentation.

Graph representation obtained by the Laplacian matrix has been used in graph drawings [16], [17].Together with the Laplacian L and the normalized Laplacian $\hat{L}$ also the matrix D^{-1}Lhas been used in clustering algorithms. According to [15] the last matrix performs best. The indexing structure of objects appearing in computer vision (and in a wide range of other domains such as linguisticsand computational biology) may take the form of a tree. An indexing mechanism that maps the structure of a tree into a low-dimensional vector space using graph eigenvalues is developed in [23]. Similar techniques have been used in [27].

D. Computer vision and pattern recognition

Spectral graph concept has been broadly implemented to resolve issues in the discipline of computer vision and pattern recognition. Examples include image segmentation, routing, image classification, and so on. These methods use the spectrum, i.e. eigenvalues and eigenvectors, of the adjacency or Laplacian matrix of a graph.

A more sophisticated idea is to represent an image's content by a graph with specially selected points as vertices. The interesting points are points in an image which have a well-defined position and can be robustly detected. Several other graphs are used. Graphs appearing in computer vision usually have a lot of vertices and simple eigenvalues [20].Techniques from spectral-graph theory have been used to develop powerful algorithms in computer vision and pattern recognition. For instance, Shi and Malik [24] have shown how the Fiedler vector (i.e. the eigenvector associated to the second smallest eigenvalue of the Laplacian matrix) can be used to separate the foreground from the background structure in images. The original procedure from [18] has been improved by using the matrix D^{-1}L(so as to maximize the normalized graph cut).

More generally, image segmentation is an essential technique in computer vision and pattern recognition. The problem is to divide the image into regions according to some criteria. Very frequently the image segmentation is obtained using eigenvectors of some graph matrices.

Graph clustering is also an important issue in computer vision and pattern recognition, since graphs can be used for the high-degree abstraction of scene and object structure. standard graph clustering methods need to clear up the correspondence problems between vertices of the original and the transformed graph what could cause computational problems. Luo, Wilson and Hancock [19], [20] have proposed spectral invariants for graph clustering.These

methods do not need to solve the vertex correspondence problem, instead they rely on using information from the spectrum of the Laplacian matrix. Finally, and we mention an application of the spectral graph theory in automatic cancer diagnosis, where the so called cell graphs are examined. They are extracted from biopsy images, and define a new set of features derived to distinguish the cancerous tissues from their healthy counterparts (see [26] for more details).

E. Internet Search:

Web search engines are based on eigenvectors of the adjacency and some related graph matrices. The most known systems are PageRank [21] (used in Google) and Hyperlinked Induced Topics Search (HITS) [22]. The structure of the Internet is represented in this context by a digraph G where web pages correspond to vertices and links between the pages (hyperlinks) to arcs. HITS exploits eigenvectors belonging to the largest eigenvalues of the matrices AAT and ATA where A is the adjacency matrix of a subgraph of G induced by the set of web pages obtained from search key words by some heuristics. The obtained eigenvectors define a certain ordering of selected web pages.

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PageRank uses similar ideas. Random walks are considered in this model. In fact, the adjacency matrix of G is normalized so that the sum of entries in each row is equal to 1. This is achieved by dividing the entries in each row by the out degree of the corresponding vertex. Equivalently, we form a new matrix P=$D_+^{-1}$where D+ is the diagonal matrix ofout-degrees. The matrix P is a transition matrix of a Markovchain and the normalized eigenvector of the largest eigenvalueof its transpose P^T defines the steady-state of the chain. Pagesare ranked by the coordinates of this eigenvector. Expository paper [23] contains a survey of both techniques.

There are many papers in computer science literature ondifferent aspects of using eigenvectors in Internet searchengines.

Theorem. If A is the adjacency matrix of a graph, then the (i,j)-entry $a_{ij}^{(k)}$ of the matrix A^k is equal to the number of walks of length k that originate at vertex iand terminate at vertex j. The same idea of ranking vertices appears with eigenvector centrality, which is also a measure of the importance of a vertex in a network. The vertex centrality is defined as the corresponding coordinate of the normalized positive eigenvector of the graph index. In fact, it assigns relative scores to all vertices in the network based on the principle that connections to high-scoring vertices contribute more to the score of the vertex in question than equal connections to low-scoring vertices. Google's PageRank is a variant ofthe eigenvector centrality measure. For more details, see [14]

F. Load balancing and multiprocessor interconnection networks:

The activity which must be executed by a multiprocessor system is divided into parts referred to as standard jobs that are given to unique processors to address them. elementary jobs distribution among processors may be represented through a vector x whose coordinates are non-negative integers.
Vector x is usually changed for the duration of the work of the system because some elementary jobs are executed even as new elementary jobs are permanently generated during the execution process. Of course, it would be ideal that the number of fundamental jobs given to a processor is the same for all processors, i.e., that the vector x is an integer multiple of the vector j whose all coordinates are equal to at least one. due to the fact this is not continually possible, it's miles reasonable that processors with a larger variety of fundamental jobs send a number of them to adjacent processors in order that the task distribution becomes uniform if viable. on this way the so called hassle of load balancing is vital in coping with multiprocessor structures. we shall gift an set of rules for the weight balancing trouble that's based at the Laplacian matrix of a graph.

Let G be a connected graph on n vertices. Eigenvalues and corresponding orthonormal eigenvectors of the Laplacian L=D-A of G are denoted by $\nu_1, \nu_2, \nu_3 \ldots\ldots\ldots\ldots \nu_n = 0$ $u_1, u_2 \ldots\ldots\ldots u_n$, respectively.
Any vector x from R^n canbe represented as a linear combination of the form x

$$=\alpha_1 u_1 + \alpha_2 u_2 + \alpha_3 u_3 \ldots\ldots\ldots\ldots\ldots \alpha_n u_n$$

By iterations

$$x^{(k)} = (I - \frac{1}{\mu} L)\, x^{(k-1)} \quad , \quad k=1,2,3\ldots\ldots\ldots\ldots\ldots\ldots\ldots\ldots(m-1) \qquad \ldots\ldots\ldots\ldots\ldots\ldots\ldots\ldots\ldots\ldots(1)$$

The number of iterations in (1) is equal to the number of non-zero distinct Laplacian eigenvalues of the underlying graph. In addition, maximum vertex degree $\triangle$ of G also affects computation of the balancing flow. Therefore, the complexity of the balancing flow calculations essentially depends on the product $m\triangle$ and that is why this quantity was proposed in [25] as a parameter relevant for the choice and the design of multiprocessor interconnection networks. See references [26], [27], [28], [29] for further information on the load balancing problem. For a survey of load balancing in wireless sensor networks the reader is referred to [30].If $m\triangle$ is small for a given graph G, the corresponding multiprocessor topology was expected to have good communication properties and has been called well-suited.

G. Anti-virus protection versus spread of knowledge

The largest eigenvalue $\lambda_1$of the adjacency matrix plays an crucial role in modelling virus propagation in computer networks. The smaller the largest eigenvalue, the larger the robustness of a network against the unfold of viruses. Infact, it was shown in [31] that the epidemic threshold in spreading viruses is proportional to $1/\lambda_1$. Another model of virus propagation in computer networks has been advanced in [32] with the same conclusion concerning $1/\lambda_1$

The virus propagation model established in [31] is a discrete time model. It uses the vector Pt $= (P_{1,t}, P_{2,t}, P_3,t, P_{4,t} \ldots\ldots\ldots\ldots P_n t)^T$ where $P_{i,t}$is the probability that the vertex iis infected at timet.

Given βand δ(where βis the virus birth rate on an edge connected to an infected vertex and δthe virus curing rate on an infected vertex), one can derive the following condition

$$\frac{\beta}{\delta} < \frac{1}{\lambda 1}$$

for elimination of viruses. We see that the network is as safer as the smaller is λ_1. We can denote the quantity $\tau = \frac{1}{\lambda 1}$ asthe epidemic threshold in spreading viruses. Hence if $\frac{\beta}{\delta} < \frac{1}{\lambda 1}$the network is safe and in the opposite case the network willbe conquered by viruses.

The intuitive explanation to this definition is that even as extra paths of a set duration we have in order to send information, we can cut up the information on these paths and coordinate them to arrive with the same number of hops at the receiver. This has the gain of equalizing source destination delays of packets that belong to the same class, which permits one to minimize the amount of packets that come out of sequence. This is applicable since in data transfers, out of order packets are misinterpreted to be lost which results not only in retransmissions but also in drop of systems throughput (see [15]).There are numerous mathematical investigations in both directions: to find graphs in particular lessons of graphs which have minimal or maximal largest eigenvalue (see, e.g. [33],[3]).

H. Statistical databases and social networks

Statistical databases are those who permit only statistical access to their records. Individual values are normally deemed confidential and are not to be disclosed, either directly or indirectly. Thus, users of a statistical database are restricted to statistical types of queries, such as looking for the sum of values, minimum or maximum value of some records, etc. Moreover, no sequence of answered queries should enable a user to obtain any of the confidential individual values. We consider here the restricted case where queries are related to the sum of values of records in the database and each record is contained in at most 2 queries. Then the query matrix corresponds to an incidence matrix of a graph G, where queries correspond to vertices and records correspond to edges. The results from [8], [10] display an interesting connection among compromise-free query collections and graphs with Least eigenvalue -2 [30]. This connection become diagonsed in the paper [9].

I. Quantum computing:

Quantum computation is a model of computation based totally on the principles of quantum mechanics although the corresponding computers have now not yet been found out. no matter the nonexistence of actual machines, the principle of quantum computingis very lots developed. For a well-known overview on Quantum information technology see, as an instance, special issue of the journal NEC research & developments [34].

It has been discovered recently [16] that integral graphs can play a role within the so called perfect state transfer in quantum spin networks. Speaking in terms of quantum physics, there is ideal state transfer between vertices of a graph if a single excitation can travel with fidelity one between the corresponding sites of a spin system modelled by the graph.

Let G be a graph with adjacency matrix A and considerthe matrix $H(t) = e^{iAt}$where tis a real variable and$i^2 = -1$. According to [24], perfect state transfer occursbetween vertices u and v of G if there is a value of t suchthat |H(t)u,v|= 1. This can happen in integral graphs but notalways.Further details on this topic can be found in [11], [24].

The 3-dimensional cube is the only connected cubic integral graph with perfect state transfer. Some other results in this direction have been obtained in [4], [5].

J. Bioinformatics

The principle of complex networks is used in bioinformatics to present biological networks, as an instance, protein-protein interplay networks [12]. In this paper eigenvectors of numerous least eigenvalues of the adjacency matrix are used to identify bipartite subgraphs in a network corresponding budding yeast.Networks performing in biology have been analyzed by using spectra of normalized graph Laplacian in [2], [3]. Spectral techniques had been carried out in reading mind networks in [9]. any other application is associated with coming across genetic ancestry, see [26].

Conclusion:

As we've got seen, spectral graph theory has numerous important applications in computer science. New applications seem very frequently. Our feeling is that cooperation between engineers and mathematicians concerning applications of spectral graph theory to computer science must be stepped forward for the advantage of both computer science and mathematics.

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